Variants of TM and
The Church-Turing Thesis

CS 154, Omer Reingold
Turing Machine (1936)

INFINITE REWRITABLE TAPE

FINITE STATE CONTROL

A N P U T
Multitape Turing Machines

\[ \delta : Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L,R\}^k \]
Theorem: Every Multitape Turing Machine can be transformed into a single tape Turing Machine.
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The Church-Turing Thesis

Everyone’s
Intuitive Notion = Turing Machines
of Algorithms

This is not a theorem – it is a falsifiable scientific hypothesis.

And it has and is still been tested
Theorem: Every nondeterministic Turing machine $\mathcal{N}$ can be transformed into a Turing Machine $\mathcal{M}$ that accepts precisely the same strings as $\mathcal{N}$.

Proof Idea (more details in Sipser)
Pick a natural ordering on all strings in $\{Q \cup \Gamma \cup \#\}^*$

$\mathcal{M}(w)$: For all strings $D \in \{Q \cup \Gamma \cup \#\}^*$ in the ordering,
Check if $D = C_0\# \cdots \#C_k$ where $C_0, \ldots, C_k$ is some accepting computation history for $\mathcal{N}$ on $w$.
If so, accept.
Recognizability via Logic

Definition: A decidable predicate \( R(x,y) \) is a proposition about the input strings \( x \) and \( y \), such that some TM \( M \) implements \( R \). That is, for all \( x, y \),

\[
\begin{align*}
R(x,y) & \text{ is TRUE } \Rightarrow M(x,y) \text{ accepts } \\
R(x,y) & \text{ is FALSE } \Rightarrow M(x,y) \text{ rejects }
\end{align*}
\]

Can think of \( R \) as a function from \( \Sigma^* \times \Sigma^* \to \{T,F\} \)

Examples: \( R(x,y) = \text{“}xy \text{ has at most 100 zeroes”} \)
\( R(N,y) = \text{“}TM N \text{ halts on } y \text{ in at most 99 steps”} \)
Theorem: A language $A$ is recognizable if and only if there is a decidable predicate $R(x, y)$ such that: $A = \{ x \mid \exists y \ R(x, y) \}$

Proof:

(1) If $A = \{ x \mid \exists y \ R(x,y) \}$ then $A$ is recognizable

Define the TM $M(x)$: Enumerate all finite-length strings $y$, If $R(x,y)$ is true, accept $\Rightarrow M$ accepts exactly those $x$ s.t. $\exists y \ R(x,y)$ is true

(2) If $A$ is recognizable, then there is a decidable predicate $R(x, y)$ such that: $A = \{ x \mid \exists y \ R(x,y) \}$

Suppose TM $M$ recognizes $A$. Let $R(x,y)$ be TRUE iff $M$ accepts $x$ in $|y|$ steps $\Rightarrow M$ accepts $x \Leftrightarrow \exists y \ R(x,y)$
Parting thoughts:

- Many natural variants of TM – all equivalent.
- Church-Turing thesis: this is not a coincidence.
- Nondeterminism doesn’t add power (unless we take efficiency into account).